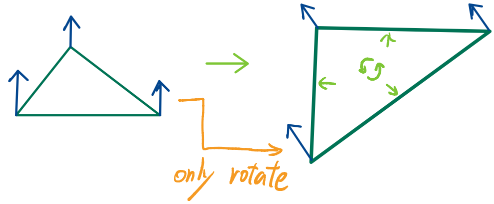
Transforming normals



① 古典幾何 : https://en.wikipedia.org/wiki/Normal_(geometry)#Transforming_normals

$$(Wn)\cdot(Mt)=0$$

$\Rightarrow (Wn)^T(Mt)=0$

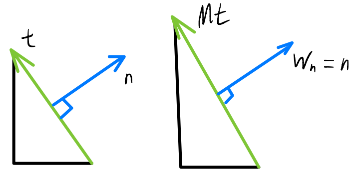
$\Rightarrow (n^T W^T)(Mt)=0$

$\Rightarrow n^T(W^T M t)=0$

$\Rightarrow n^T(W^T M)t=0$

If $W^T M = I \Rightarrow n^T t = 0 = n\cdot t = 0$

$\Rightarrow W^T = M^{-1} \Rightarrow W = (M^{-1})^T$



不過因為用"=0"來算，所以 scale 就無效了，

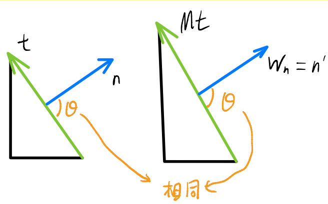
② https://paroj.github.io/gltut/Illumination/Tut09%20Normal%20Transformation.html

R is pure-rotation $\Rightarrow R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $R^T R = \begin{bmatrix} (\cos\theta)^2+(\sin\theta)^2 & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & (\sin\theta)^2+(\cos\theta)^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow R^T = R^{-1}$

when S is pure scale a times $\Rightarrow S = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, $S^T = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = S$



for 3x3 $M = R_1 S R_2$

(R_1, R_2: pure-rotation; S: scale)

$(A^T)^{-1} = (A^{-1})^T$
$(AB)^{-1} = B^{-1}A^{-1}$
$(AB)^T = B^T A^T$

$W = R_1 S^{-1} R_2 = (R_1^T)^T (S^T)^{-1} (R_2^T)^T = (R_1^{-1})^T (S^{-1})^T (R_2^{-1})^T$

$= (R_2^{-1} S^{-1} R_1^{-1})^T = ((R_1 S R_2)^{-1})^T = (M^{-1})^T$

we will normalize this anyway (in fragment shader) this is a "don't care"